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TRAINING FOR EFFICIENCY IN ELEMENTARY MATHEMATICS.

By E. H. Koch, Jr.

To-day the dominant note in education is efficiency. It is fitting therefore that we should consider the factors for training in efficiency in elementary mathematics. The work of this association like that of kindred societies affords the best opportunity for promoting efficiency. In this respect our work does not differ from the main work of every other world movement as manifested in the conventions and meetings of men with their coworkers. In fact the agitations for improvement in the teaching of mathematics is as you know but one phase of a perplexity which confronts every field of human activity.

The expressions of opinions, serious comment, great thoughts, inventions, social and intellectual achievements are not spontaneous nor do they occur in a single mind because they are results of race development. How often in our experience have we been spared from expostulating on some cherished idea, design, method or invention because accident has given priority to some other independent worker. Such first announcements are apt to be vague and undefined and as a consequence men take issue and often give way to passionate disregard and intolerance for the tenacious attitude of an adversary.

I have a pretty firm belief in that greatest of paradoxes, viz., that both sides of an argument are right. Therefore, my friends, I am going to regard your assertions relating to the cultural and utilitarian value of any mathematics material as amphibolous aspects of an ideal future vocation. I am not going to reconcile these aspects, because they need no reconciliation. If you will permit me to state an analogy it appears to me as if some of us had gone into a long glass tube while the others remained outside. In so doing we have chosen our limitations of vision. When we recognize the fragile barrier that separates our groups we can understand why we look upon one another telescopically or microscopically. We owe to a heritage the fallable belief in a dualism which has been symbolized constantly in the development of our literature, art, religion and science. The sense of opposites expressed as north and south, hot and cold, life and evil suggest culture and utility. Let us give praise to that noble soul who first annunciated the law of relativity. Now you know there is no greater avenue of adventure than higher geometry which reveals the infinite attenuation of life and comprehends the processes which guide the unborn possibilities of the future.

The isolation of the subjects of the old curricula has produced sterility.

"Education has not been able to keep abreast with the innumerable problems of our complex life very largely because its emperical methods are obsolete."

"Even scientific men are recognizing the fact that they must apply scientific methods to determine the circumstances which promote or hinder the advancement of science."* It is surely time for educators to apply the scientific method to determine those factors which enter into the problem of the efficiency of educational systems and more particularly to the special problem regarding the training for efficiency in elementary mathematics.

The best definition of efficiency is borrowed from the technical man who first ascribed it in a commercial way to a machine. Efficiency is the ratio of net output to gross input.

$$(a) \qquad \text{Efficiency} = \frac{\text{output}}{\text{input}}.$$

The human being is also a machine in the sense that it is a highly organized complexity of more or less coördinated parts. The intellectual attributes when associated with the nervous system bear to the framework of the body the analogous relation that the forces of nature bear to structural material.

Efficiency is a measure and is therefore expressed as a ratio between a realized performance and an ideal possibility. There are as many different aspects of efficiency as there are considerations in any mental or physical situation. When you speak of health or strength you unconsciously give efficiency a special name.

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* *Science,* November 4, 1910.

It was quite natural to expect technical men to apply this term to the situation which presents itself in technical and industrial education, because the facts in their case were more definitely known. In this respect they have made a beginning.

It is my purpose to-day to indicate how such an application can be made in the special field of mathematics. Let us consider the summary of our technical colleagues who volunteer the following meanings of efficiency applied to education:—*

1. The Efficiency of the Curriculum—the ratio of the useful information and mental training offered by a college to that which an ideal college should offer

$$(1)\quad E_C = \text{Efficiency of curriculum} = \frac{\text{what is offered}}{\text{what should be offered}}.$$

2. The efficiency of the teacher—the ratio of the amount of knowledge and mental development acquired by the student to the amount of the opportunities offered by the college

$$(2)\quad E_1 = \text{Efficiency of teacher} = \frac{\text{what and how he teaches}}{\text{what and how he should teach}}.$$

3. Efficiency of the student as receiver—the ratio of that which the student assimilates to that which the college offers.

$$(3)\quad E_R \ \text{Efficiency of student as receiver} = \frac{\text{what he assimilates}}{\text{what the college offers}}.$$

4. Efficiency of the student as giver—the ratio of the use made by the student of his college education during his professional career; to the potential energy stored in him at the time of graduation.

$$(4)\quad E_G = \text{Efficiency of student as giver} = \frac{\text{what he gives to the country}}{\text{what he has received from the college}}.$$

There was a time when the soldier was the most efficient man in the state. Centuries later the mantel of efficiency befel to the lot of the lawyer. The evolution of the political, social and industrial progress in the development of civilization compels the finger of time to point to the men of science as the dominant leaders of the nations.

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* Karapetoff, Contributions to the discussion of efficiency in engineering education, *S. P. E. E. Bulletin,* Vol. XVIII.

"In America at least," says Professor Townsend,* "we have come to accept as a fundamental principle that the supreme test of an education is the efficiency of the training it gives the individual to meet the demands of organized society and at the same time enable him to contribute most directly or indirectly to the general progress of national life." Therfore:

$$(5)\quad E_M = \text{Man's overall efficiency} = \frac{\text{his productiveness} + \text{his contributions to society}}{\text{latent possibilities times education in its larger sense}}.$$

$$(6)\qquad E_M = \frac{P+C}{LS}.$$

P = productiveness.
C = contributions to society.
L = latent possibilities.
S = education in its larger sense.

Each one of these quantities is resolvable into innumerable factors many of which although not definitely known are nevertheless familiar bywords in the parlance of educational meetings. The unknown factors are equally important and must be determined and investigated. The task at first thought appears so stupendous as to seem almost insoluble, but if we apply the scientific spirit we need not despair. Some years ago the government sought to devise an automatic machine for predicting the tides for any locality, and for long stated intervals in advance. The question was carefully undertaken, although the obstacles seemed as insurmountable as the elements entering into weather prognostications. The working equation contained thirty-five variables. To-day that machine is a realization. Our problem narrows down when we confine our attention to the elements of mathematics which contribute to the efficiency of the individual.

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* *Science,* November 4, 1910.

The individual may be likened to the incandescent lamp which has an efficiency of approximately one per cent. This means a waste of ninety-nine per cent. of the energy stored in the coal. Briefly told the final efficiency is the product of all the contributing efficiencies involved in the mechanical and electrical transformations and transmissions through boiler, engine, generator, wires and filaments of the lamp.

$$\text{(A)}\quad E_S = \text{Efficiency of the system} = \frac{\text{energy given out as light}}{\text{energy stored in the coal}}.$$

$$\text{(B)}\quad E_S = \frac{\text{energy stored in the coal} - \text{total losses of energy}}{\text{energy stored in the coal}}.$$

In like manner man's overall efficiency may be expressed in terms of losses.

$$(8)\qquad E_M = \frac{LS - \{O + I + F\}}{LS}.$$

substituting in (6) and (B).

O = lost opportunities,

I = impediments,

F = failures,

$$(9)\qquad \therefore P + C = LS - (O + I + F),$$

substituting in (8) (6).

Or we can represent man's overall efficiency as the product of all the efficiencies in the educational power station.

$$(10)\qquad E_M = E_C E_T E_R E_G. \quad \text{See (1) (2) (3) (4).}$$

We are especially interested in the mathematics applied to E_C. We shall define mathematics efficiency as the ratio of the factors which contribute to man's social, political and industrial abilities to the stimulus and power given by mathematics training.

$$(11)\quad E_m = \text{mathematics efficiency} = \frac{\text{factors which contribute to social, political, industrial abilities}}{\text{stimulus and power given by mathematics training}}.$$

$$E_m = \frac{X}{M}. \tag{12}$$

$$X = f(x_1 x_2 x_3 x_4 \ldots x_n), \tag{13}$$

$$M = \phi(m_1 m_2 m_3 \ldots m_k), \tag{14}$$

$$\therefore E_m = \frac{f(x_1 x_2 x_3 \ldots x_n)}{\phi(m_1 m_2 m_3 \ldots m_k)}. \tag{15}$$

$x_1 x_2 x_3 \ldots x_n =$ the factors affected by mathematics which contribute to man's social, political and industrial abilities,
$X =$ a function of $x_1 x_2 \cdots x_n$,
$m_1 m_2 m_3 \ldots m_4 =$ the potential contributions of algebra, geometry, analysis and mechanics,
$M =$ a function of $m_1 m_2 \ldots m_k$.

There are two ways of increasing E_m: either increase the numerator X or decrease the denominator M.

The known m's are usually summarized as:

$m_1 =$ the notions of number, measure, form, element,
$m_2 =$ the focusing of law through symbols and graphics,
$m_3 =$ the visualization and interpretation of law through formulas and graphic representation,
$m_4 =$ the appreciation of coexistences and sequences of phenomena,
$m_5 =$ the projection of the intellect beyond experience,
$m_k =$ not defined.

We shall next turn our attention to the x's and challenge their validity. We have been talking about these x's for a long time and I dare say have invented a few out of mere defense for an educational hobby. We must not forget one of the staunchest and earliest critics, Professor Perry,* " who holds that the study of mathematics began because it was useful, it continues because it is useful and it is valuable to the world because of the usefulness of its results in

$x_1 =$ producing the higher emotions and giving mental pleasure,
$x_2 =$ brain development,

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* British Association Meeting, Glasgow, 1901.

x_3 = producing logical ways of thinking,
x_4 = aid given by mathematical weapons in the study of physical science,
x_5 = passing examinations,
x_6 = giving men mental tools as easy to use as their legs or arms, enabling them to go on with their education,
x_7 = teaching a man the importance of thinking things out for himself and so delivering him from the present dreadful yoke of authority,
x_8 = making men... feel that they know the principles,
x_9 = giving acute philosophical minds a logical counsel of perfection altogether charming and satisfying."

"Any subject may be useful as applicable to some special purpose or need of life or it may be useful as affording valuable mental discipline."

The following x's were excerpted from various sources:

x_{10} = mathematics should promote culture, which means the subjugation of imitative power to the creative so as to make the individual develop centrifugal force, individuality, critical opinion and transform that which is read into conversation and life.*
x_{11} = mathematics should maintain the standards of a liberal education which implies a sense of truth and beauty.
x_{12} = mathematics should promote the identical spirit of science and literature which is to transform something of value from the unknown into the realm of the known.
x_{13} = mathematics should respond to the dominant activities of the nation.
x_{14} = mathematics must justify itself in the acquisition of information of commercial value.
x_{15} = mathematics should provoke intellectual tolerance—give breadth of thought.
x_{16} = mathematics should stimulate enthusiasm.
x_{17} = mathematics should instill courage and sympathy with life.
x_{18} = mathematics should give a realization of the value of human life as a public asset.

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* Address Professor Aston, Columbia University, September 28, 1910.

x_{19} = mathematics should promote intelligent leadership in matters of public concern.
x_{20} = mathematics should promote economy of time and effort.
x_{21} = mathematics should promote diligence, conscientiousness.
x_{22} = mathematics should promote strength of character.
x_{23} = mathematics should promote judgment.
x_{24} = mathematics should promote fidelity, poise.
x_{25} = mathematics should promote neatness and accuracy.
x_{26} = mathematics should train in habits of attention.
x_{27} = mathematics should train in system in dealing with material.
x_{28} = mathematics should train in clear and interesting presentation both oral and written.
x_{29} = mathematics should train for foremanship, management and other executive positions.
x_{29} = mathematics should render service in the development of the national resources in aiding the growth and expansion of its industries and of its commercial power and in the conservation of the resources that constitute the inherited wealth of a people.*
x_{30} = mathematics should furnish a solid foundation on which the more advanced work of college and technical school may be based.
x_{31} = mathematics should develop an appreciation of the methods and spirit of pure science.
x_{32} = mathematics should lead to precision.
x_n = last but not least mathematics should promote common sense.

There are many more factors of the numerator which may be mentioned. The critical test must be applied to each x, the so-called desirable accomplishment, to see if it rings true or whether it has been coined to defend in a vague way some pet fancy of the mathematics department. Is it not generally true that in determining the efficiency of any branch of study these same accomplishments are more or less in advocacy. It seems to me that if this were true we would be wasting effort and time in accomplishing in an inefficient way what may be more efficiently accomplished in another subject.

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* See Science and Public Service Address University of Illinois Biologic Station, July 22, 1910. "American Educational Defects," *Science*, October 28, 1910. Report of Training of Mathematical Instructors, *Bulletin A. Math. Soc.*, November, 1910.

Beyond the ordinary claims mentioned which have been advanced and reiterated is the potent claim of a very broad consideration. I am convinced that in every department of human investigation there are definite underlying mathematical relations which are either little understood or totally unknown.

It was a great pleasure for me some two years ago to listen to a score of weekly addresses delivered by one of the representatives of each of twenty-four departments of study at Columbia University. In the period of one hour at the disposal of each of these scholars he gave a brief summary of the historical development of his subject with the emphasis on its important contributions to mankind and a hint of future anticipations. I do not believe I have listened to any other series of lectures with a like interest, not because I fully understood their comprehensiveness, but because of the acute sense of the mathematical basis and generalizations underlying these subjects. If you will permit me to be a little more personal I would state that it is a phase of investigation that I have been interested in for a number of years and I owe my inspiration to our esteemed colleague Dr. Schwatt.* It was extremely encouraging to have so many of these representative men acknowledge at least a meagre indebtedness to the mathematical elements in their subject. Someone has said "a man's capacity for mathematics is paralleled with the development of the mathematical activity present in the great departments of investigation." Formerly the subject, biology, was considered remotely distinct and separated from mathematical analysis. Then those students who were not inclined to the mathematical discipline of their courses flocked to biologic studies only to learn that they had chosen intensely mathematical subjects. It is for this reason that most of our biologic activities are scarcely more than the clerical work of classification and cataloging.

It seems to me that our next great work as mathematics teachers will be to lend our influence to the discovery of the relationships which underlie other fields of research. If these relationships belong in any mathematical category as I believe they do then we have a large problem whose solution shall not only conserve the mathematical field but shall add the greatest stimulus to mathematics training.

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* "Some Considerations Showing the Importance of Mathematical Study," Philadelphia, 1895.

Let us determine in how far each of the arbitrary divisions of the subject matter of mathematics contributes to desirable human achievements. Let us give proportionate weights to these divisions which we designate as algebra, geometry, analysis and mechanics and determine whether the time spent upon them is justified or whether the raw product, the material of our text-books, is poor or whether we are extravagant or indulgent in the luxuries of our subject. It is generally conceded that mathematics knows no poverty. If therefore we should discover that our arbitrary division of mathematics is wrong let us have the good sense to change it, and if we have the conviction that the subject itself is wrong let us have the courage to throw it out. We must recognize the cold hard fact that we are dealing with a commercial problem in which we shall be harder pressed. If we can give no adequate answer then we are no better than the unskilled laborer in the street. The tendency in education is democratic not aristocratic, as men have vainly attempted to make it in the past. Our ranks are swollen with men and women whose ideals and culture are no higher than those of despised workers in other vocations. Professional life is no longer a guarantee of the homage of exaltation. Why is it? Because we have introduced so much in our studies that is unreal. We have deceived ourselves into believing that our efficiency was measured by the number of students graduated or passed to a higher grade. What about all those young men and women who have been neglected because we thought their withdrawal from school or college was an expression of unfitness. Yes, an unfitness in most cases for an inefficient school system. Now as mathematics teachers how heavily have we played in this rôle. We must not be amazed then if we hear a challenge put to our cherished subjects. Will algebra, geometry, trigonometry, analytics, calculus, even arithmetic, stand the test when we ask without bias are they taught efficiently in their present text-book form. They have served us well in the past but had we not better tear them down and build new structures in their places? If we are content to leave them remain for a longer time we must repair and replenish them. Most of our teachers look upon algebra, geometry and trigonometry as if they were the body of materials embodied in a standard text-book on that subject. These are the halt but we have just as many blind teachers who regard the formula as wooden as if it were intended merely for substitution from data instead of for its larger purpose of interpretation. These impediments to efficiency are the result of our failure to recognize the world as a domain of relationships. It does seem timely that we should incorporate these subjects and teach them more efficiently under one organized whole. I believe we can teach mathematics more efficiently by presenting theory through the problem and postponing it until we need it and can make immediate use of it. In this way we can avoid a great amount of the drudgery of algebra. I see some of you staggering under the hints of corporate aggrandizement. Nothing of the kind, my friends. It is simply the motherlove of mathematics applied to the correction of the errors of her brood.

"As each subject grows it fringes" and in the garden of mathematics there are weeds to pull. One of the most persistent weeds that grows among the flowers in our estate is the so-called "problem." Why do we permit these monstrosities, these unreal problems to fill our text-books? They represent a type of fiction which is revolting to the child. They are vicious in principle because they contradict the realities of life. They negate that highest virtue of mathematics, the search for truth. We defined efficiency as a ratio, a fraction. Every puzzle, or "faked" or "unreal" problem inserted in our text-books increases the denominator of our fraction and therefore decreases the efficiency. We should establish the principle never to insert a problem which has to be worked backwards in contradistinction with the realities of life. There is plenty of good material for real live problems but if our knowledge of real things is too limited we had better omit the fictitious problems entirely. I am reminded of a problem in a recent text-book which must have been written during a house cleaning epidemic. "My window is a yard wide. From a brass curtain rod I cut off one sixth of its length and finding it still too long cut off one fifth of the length remaining. But I had to cut it the third time, taking off one tenth of its length to make it fit. What was the length of the rod?" Now no sane person in the world would ever be concerned about such a shade proposition. I tell you, ladies and gentlemen, our mathematics must be diseased when we have to insert such stuff. To make ourselves at home let us refer to the problems given by the College Entrance Examination Board in June, 1910. Having been a mathematics reader for the same board for eight years I am loath to criticize excepting that it be interpreted in the light of helpfulness, justified by experience. Mathematics Paper AI contained two problems. (1) "An estate of $36,500 is divided among 5 men, 4 women, and 6 children. A man and a child together get as much as 2 women. Four children together get as much as a man and a woman together. Find how much each man, woman, and child, respectively, receives; if all men are treated alike, all women alike, and all children alike." Think of that problem given in a democratic country. This is an obsolete and vicious problem. It is not constructive but destructive in so far as it negates common law and truth. It had better be omitted and thereby give the paper a better moral tone. We haven't time for puzzles in a crowded curriculum. The second problem reads: "A motor car traveled 20 miles at a certain uniform rate and then returned over the same route at a different uniform rate, the running time for the round trip being 2 hrs. 15 minutes. One third of the outward trip and one half of the return trip together occupied 55 minutes. Find the two rates of traveling." This is not a "real" problem but is a redress in modern equipment of an ancient text-book puzzle. Nobody would ever meet a "problem" of this kind and therefore it is a "type" that causes our students to lose respect for their mathematics. The biggest fool on earth wouldn't run "a motor car at a certain uniform rate and then return at a certain different uniform rate!" and the fractions of each trip would never disturb the calculating minds of the greatest or least mathematicians who rode in such a car.

Ladies and gentlemen, I maintain that it is a disgrace in this time and age to continue to suffer under this yoke of authority and I demand in the name of the thousands of students who are preparing for college and especially the technical school that you give a voice of protest against this infliction.

There has been reason for honest variance of opinion regarding the cultural and utilitarian value of mathematics as long as these claims remained entrenched in the old undisputed doctrines. "There is no subject labelled cultural which is not at the same time practical and there is no subject labelled practical which is not at the same time cultural." Through mutual understanding we realize that education is beginning to teach subject matter in terms of daily life and is recognizing the factors which are of value to our people. We have made the first serious attempt in the history of American education to meet the demands of our economic, industrial and social problems. We have learned that tremendous industrial forces have been developed without any commensurate adjustment of our school system. We may no longer disregard the significance of industry in its influence upon our life. Our systems and subject matter must be rehabilitated. Out of the prevailing systems there have emerged forms of practical mathematics which connect the school with the industries and vocations of our people. They involve a more or less special training which is best adapted to make the student discover himself and to promote the qualities essential to his success in a specific vocation. We are getting a little closer to the various types of mind and the precise factors of intelligence.

Two years ago the speaker undertook the unification of the essentials of elementary mathematics for electrical students. The subject matter of this practical mathematical extends over the usual range of elementary mathematics including the notions and symbolism of the elements of calculus and vector analysis and vector applications. This was necessary in order to meet the students' requirements in both direct and alternating current work. All non-essentials are omitted and the teaching of theory is postponed until the student requires it in his work. We count this a great gain in economy. The student has a clear understanding of his work. He can express ideas intelligently. He has the power of applying knowledge practically. He realizes the right thing to do in an emergency. He accomplishes a reasonable amount of work in a definite time. His class room is always a work room. His work is interesting. It gives him pleasure. He has greater physical endurance, poise, displays greater fidelity, assurance, neatness, accuracy and the greatest of all blessings, common sense. He is also trained for leadership and executive positions by imposing upon him responsibility of foremanship work in the class room, laboratory and shop. Upon entering the class room he immediately reports his outsigned work on a time sheet on the front cover of his work-book and makes a similar entry at the close of the period. The work-book contains every scrap of work which the student has completed and this book is retained by the instructor who has the students' entire work under constant supervision and inspection. The work of the instructor is made more efficient by the coöperation of the students through their foreman. We are simply recognizing a well-established principle that a man can learn to direct others by first learning to be directed by others. Secondly he can learn to guide others best by working with them, sharing their trials and disappointments and gaining an intimate appreciation of and sympathy with their modes of thought and action. "Of the many evils which our examination system has inflicted upon us the central one has consisted in forcing our school and university teaching into moulds determined not by the true interests of education but the mechanical exigencies of the examination syllabus."*

The time is not far distant when our college and state examination boards will recognize these facts and they will be compelled to offer examinations in a unified course such as practical mathematics. These demands will come first from the technical schools.

We cannot all be great mathematicians, and so our universities would become more efficient if they did not attempt to duplicate work which could be undertaken better by a few institutions. I wish we would hear the phrase institutions for teaching even more often than we hear about institutions for learning. Would it not be the part of wisdom if many of our institutions devoted their time in discovering how mathematics is contributing to other fields of intellectual activity and thereby promote such projects. We have seen an increased efficiency in mathematics teaching since history stepped in and opened a new pathway. We should establish experiment stations not only in agriculture and engineering, but also in mathematics and language and other subjects as well. Then the study of higher mathematics will have an immediate justification as well as blazing the future pathway of knowledge.

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* *Science*, September 23, 1910.

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COMPANIONSHIP.

But surely it is no very extravagant opinion that it is better to give than to receive, to serve than to use our companions; and above all, where there is no question of service upon either side, that it is good to enjoy their company like a natural man.—*Robert Louis Stevenson.*